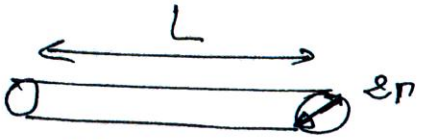
TD5 | Diffusion de molécules à travers une membrane

III

1) $\vec{J}_n = -D \underbrace{\overrightarrow{\text{grad}}\, n_v}_{m^{-4}} = -D \frac{\partial n_v}{\partial x} \vec{e}_x$ → densité particulaire (m^{-4})

coefficient de diffusion ($\frac{s^{-1} m^{-2}}{m^{-4}} \equiv s^{-1} m^{+2}$)

vecteur courant volumique de particules ($s^{-1} m^{-2}$)

2) 

$\mathcal{S}_p = \pi r^2$

$$\Phi_p = \iint_{\mathcal{S}} \vec{J}_n \cdot \vec{d\mathcal{S}} = \iint_S \vec{J}_n \cdot \vec{n}\, dS$$

$$\Phi_p = \iint_S -D \frac{\partial n_v}{\partial x} (\vec{e}_x \cdot \vec{n})\, dS$$

$\vec{n} \equiv -\vec{e}_x$ $\quad \Phi_p = +D \frac{dn_v}{dx} \iint_S dS$

$$\Phi_p = D \frac{dn_v}{dx} \pi r^2$$

$$\frac{dn_v}{dx} \simeq \frac{n_{2v}(t) - n_{1v}(t)}{e} = -\frac{\Delta(t)}{e}$$

$$\Phi_p \simeq -D \frac{\Delta(t)}{e} \pi r^2 \Rightarrow .$$

3) $\Phi_m = p \times S \times \Phi_p = -\frac{D \pi r^2 p S}{e} \Delta(t)$

$= -K \Delta(t) S$ avec $K = \frac{p \pi r^2 D}{e}$

$$\Rightarrow r = \sqrt{\frac{eK}{p\pi D}} = \sqrt{\frac{10^{-5} \times 10^{-6}}{10^{10} \times \pi\, 10^{-9}}} = 0{,}56\,\mu m$$

4) $$-\frac{dN_1(t)}{dt} = \frac{dN_2(t)}{dt} = \phi_m = -KS\Delta(t)$$

5) $$\frac{1}{V_1}\frac{dN_1(t)}{dt} = \frac{dn_{1V}(t)}{dt} \qquad \frac{1}{V_2}\frac{dN_2(t)}{dt} = \frac{dn_{2V}(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{d\left(n_{1V}(t) - n_{2V}(t)\right)}{dt} = \frac{dn_{1V}(t)}{dt} - \frac{dn_{2V}(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{1}{V_1}\frac{dN_1(t)}{dt} - \frac{1}{V_2}\frac{dN_2(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{KS(\Delta(t)}{V_1} + \frac{KS\,\Delta t)}{V_2} = KS\,\Delta(t)\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$$

$$\boxed{\frac{d(\Delta t)}{dt} - \frac{\Delta(t)}{\tau} = 0}$$

$$\frac{d(\Delta t)}{\Delta(t)} = -\frac{dt}{\tau} \Rightarrow \Delta(t) = C\,e^{-\alpha t}$$

$$\alpha = \frac{1}{\tau}$$

$$\Delta(t=0) = \Delta_0 = C$$

$$\Delta(t) = \Delta_0\, e^{-t/\tau}$$

t_1 Tq $\quad \Delta(t=t_1) = \frac{\Delta_0}{10} = \Delta_0\, e^{-t_1/\tau}$

$$\Rightarrow e^{t_1/\tau} = 10 \Rightarrow t_1/\tau = \ln 10$$

$$t_1 = \tau \ln 10 = 2{,}3\,\tau$$